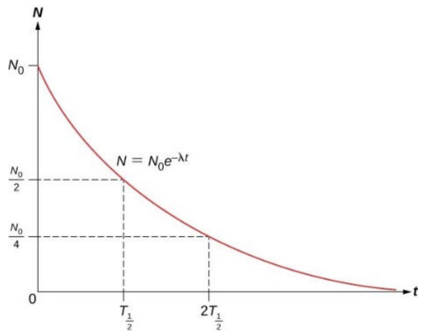
RADIOACTIVITY

DECAY LAW

No is the number of radioactive atoms/Nuclei at t=0
N is the number of radioactive atoms/Nuclei left after time t
Larger the value of N larger the decay
Let dN be the number of decays in time dt



$$dN \;\alpha\; -N\,dt$$
$$dN = -\lambda\, N\, dt$$

where λ is constant of proportionality called the decay constant.
The negative sign shows that the change in nuclei dN is negative.

$$\frac{dN}{N} = -\lambda\, dt$$

$$Integrating\ we\ get, \int_{No}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$[\ln N]_{No}^{N} = -\lambda[t]_0^t$$

$$\ln N - \ln No = -\lambda\,[t-0]$$

$$\ln \frac{N}{No} = -\lambda t$$

$$N = Noe^{-\lambda t}$$

This is the decay law or law of radioactive decay.

ACTIVITY

Activity A, is rate of disintegration or disintegrations per unit time

$$A = -\frac{dN}{dt} = -\lambda\, N = -\lambda\, No\, e^{-\lambda t}$$

$$A = Ao\, e^{-\lambda t}\ where\ Ao = -\lambda No$$

Activity is measure in becquerel (Bq) in SI unit
One becquerel is one disintegration per second
1 Curie (Ci) = 3.7 x 10^{10} Bq

HALF LIFE

The time taken for the number of parent radioactive nuclei of a oarticular species to reduce to half its value is called its half – life ($T_{½}$)

$$N = No\, e^{-\lambda t}$$

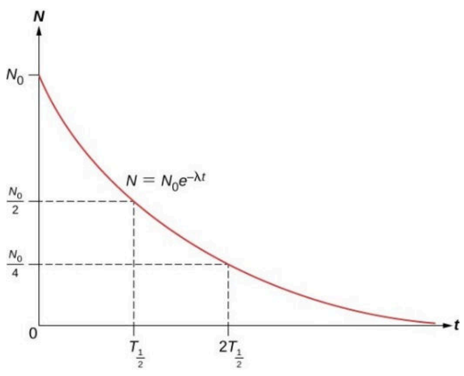
$$Put\ N\ as \frac{No}{2}$$

$$\frac{No}{2} = No\, e^{-\lambda T_{\frac{1}{2}}}$$

$$2 = e^{\lambda T_{\frac{1}{2}}}$$

$$\ln 2 = \lambda T_{\frac{1}{2}}$$

$$T_{1/2} = \frac{0.693}{\lambda}$$



So in time t=$T_{½}$ the radioactive parent nuclei reduce to No/2
In time t=2$T_{½}$ the radioactive parent nuclei reduce to No/4 and so on

AVERAGE LIFE

Average or mean life = $\frac{\text{Total lifetime of all nuclei}}{\text{Total number of nuclei in sample}}$

$$\tau = \frac{\int_{No}^{0} t\, dN}{\int_{No}^{0} dN} = \frac{-\lambda\, No\, \int_0^\infty t\, e^{-\lambda t}\, dt}{[N]_{No}^{0}}$$

$$\tau = \frac{-\lambda\, No\, \int_0^\infty t\, e^{-\lambda t}\, dt}{[0 - No]} = \lambda \int_0^\infty t\, e^{-\lambda t}\, dt$$

On integrating we get

$$\tau = \frac{1}{\lambda}$$

Thus, decay constant is also the reciprocal of mean or average life of the radioactive species.